Avoiding a multiversal apocalypse from Coleman–De–Luccia transitions

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Work in progress with B. Hassfeld, B. Freivogel and A. Hebecker

The multiverse

- String multiverse [Bousso, Polchinski '00][Denef, Douglas '04][...] needs **dynamical transition rates**.

- Most events occur on very large late-time slices

Late-time Lorentzian picture?

Euclidean [Coleman '77][Callan, Coleman '77][Coleman, De Luccia '80]

Hamiltonian [Fischler, Morgan, Polchinski '90][De Alwis, Muia, Pasquarella, Quevedo '19][Cespedes, De Alwis, Muia, Quevedo '20][[Cespedes, De Alwis, Muia, Quevedo '23]

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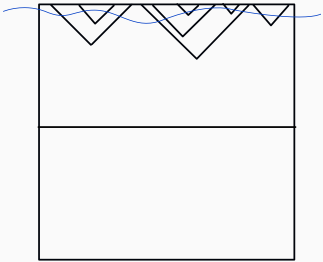

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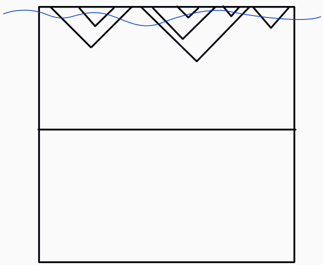

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Coleman–De–Luccia (CDL) [Coleman, De Luccia ’80]

$$S_{\mathrm{E}} = \sum_{i=1}^{2} \left[M_{\mathrm{p}}^2 \int_{\mathcal{M}_i} \sqrt{g_i} \left(-\frac{\mathcal{R}_i}{2} + 3H_i^2 \right) - \int_{\partial \mathcal{M}_i} M_{\mathrm{p}}^2 \mathcal{K}_i \right] + \int_{\partial M_1} \sigma$$

- Pure tension thin wall between two de Sitter vacua:

$$H_1,\ H_2, \qquad \xi \equiv \frac{\sigma}{2M_{\mathrm{p}}^2}.$$

- Instantons: Two four-sphere patches glued on an S^3 wall

- **Wall radius is fixed by Israel junction condition:**

$$R_{\mathrm{crit}}^2 = \frac{4\xi^2}{(H_1^2 - H_2^2)^2 + 2\xi^2(H_1^2 + H_2^2) + \xi^4}$$

$$\boxed{\Gamma \sim e^{-B_{\mathrm{CDL}}}, \quad B_{\mathrm{CDL}} = S_{\mathrm{E,instanton}} - S_{\mathrm{E,vacuum}}}$$

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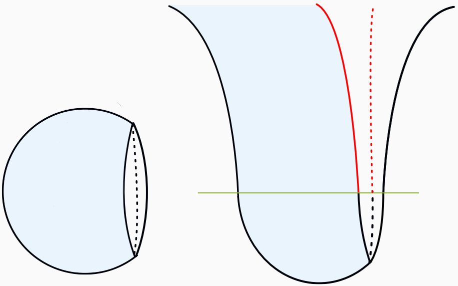
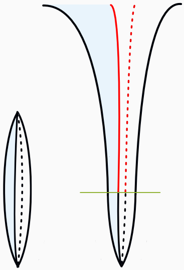


Type A

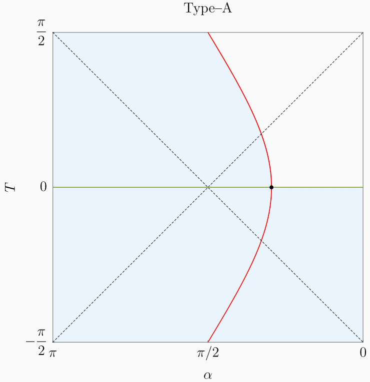


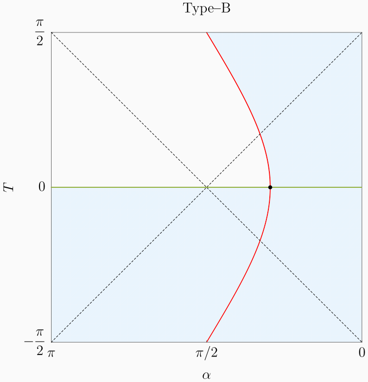
Type B

$$x = \frac{\xi^2}{H_1^2 - H_2^2} < 1$$

$$x = \frac{\xi^2}{H_1^2 - H_2^2} > 1$$

Penrose diagrams





- Type–A: down-tunneling can be understood inside a parent static patch
- Type–B (or up-tunneling): naive waist continuation says that **most of the parent spatial sphere disappears**

The Larfors–Johnson problem

[Johnson, Larfors '08]×2 [Aguirre, Johnson, Larfors '10]

- In type IIB flux Landscape: $m_{\text{flux}} \sim \frac{1}{\mathcal{V}_0}$
- Second step: Kähler stabilization

$$m \ll m_{\text{flux}} \qquad \Longrightarrow \qquad m\mathcal{V}_0 \ll 1$$

- Flux changing domain wall are D5/NS5-branes with tension

$$T = \frac{A_0}{\mathcal{V}} = \frac{A_0}{\mathcal{V}_0} e^{-B_0 \phi}$$

- In KKLT and LVS [Kachru, Kallosh, Linde, Trivedi '03][Balasubramanian, Berglund, Conlon, Quevedo '05][...] large tension ⇒ **Type–B regime**:

$$\underline{\text{KKLT:}} \quad x \sim \frac{1}{|W_0|^2} \gg 1\,, \qquad \underline{\text{LVS:}} \quad x \sim \mathcal{V}_0 \gg 1$$

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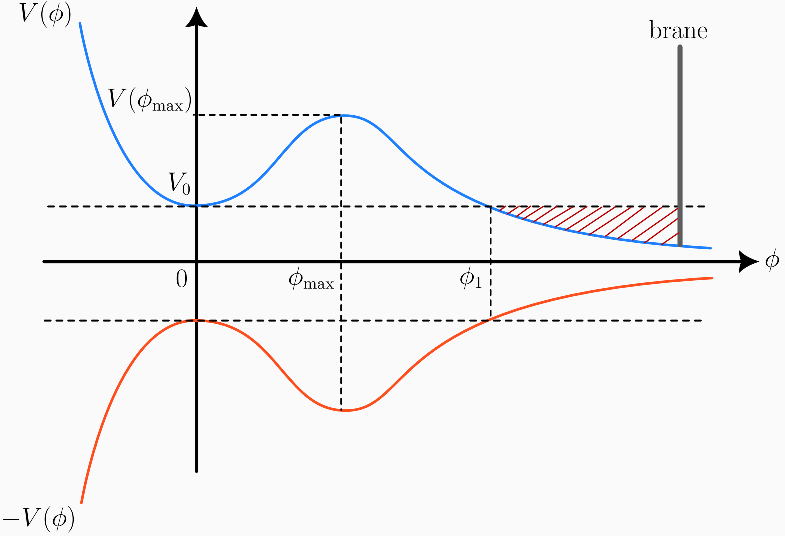
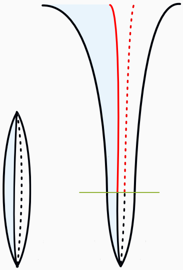
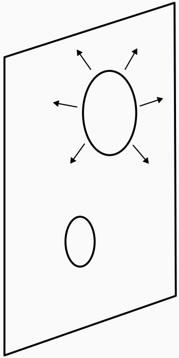


Type–B + Unstable domain wall + CDL = Apocalypse



Type B

$+$



$\longrightarrow$ **Crunch of multiverse**

Late-time Lorentzian strategy

- Global dS slicing:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{\cosh^2(Ht)}{H^2}\mathrm{d}\Omega_3^2$$

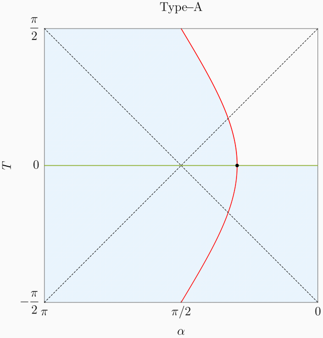
- **Unique set of solutions**
  [Cespedes, De Alwis, Muia, Quevedo '20]

- One quantum variable: Wall radius

$$R(t) = a(t)\sin\alpha(t).$$

- WDW/WKB Hamiltonian problem

$$\Gamma = e^{-B}\,, \qquad B = 2\int_0^{R_{\mathrm{on-shell}}} \mathrm{Im}\, p_R\, \mathrm{d}R$$



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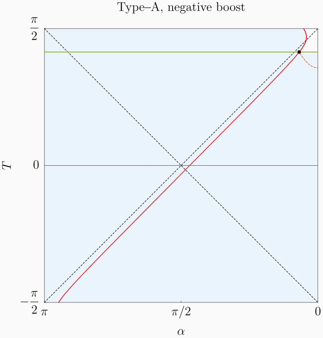

Small off-shell bubble grows inside an already large parent dS

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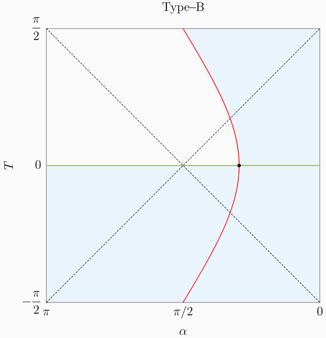

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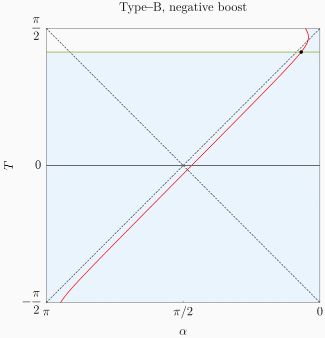

Not a small bubble

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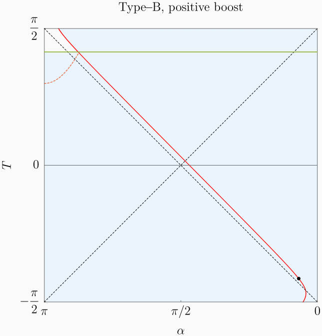

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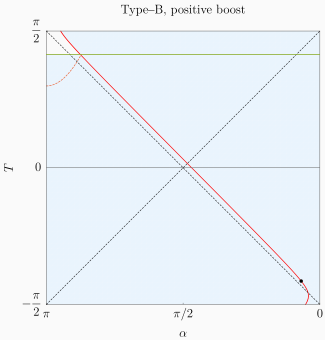

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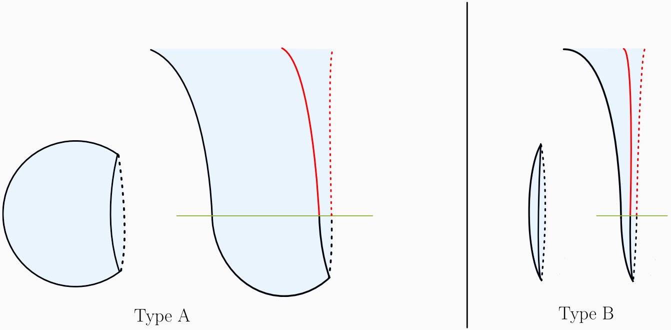
Bubble of Nothing (BoN) [Witten '82][...]



- Replace second vacuum by an end-of-the-world (ETW) brane
- Only one dS side with Hubble factor H

$$S = M_{\mathrm{p}}^2 \int_{\mathcal{M}} \sqrt{-g}\left(\frac{\mathcal{R}}{2} - 3H^2\right) + \int_{\partial\mathcal{M}} M_{\mathrm{p}}^2 \mathcal{K} - \int_{\partial M} \sigma$$

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- With $s \equiv \dot{R} - HR$:

$$\boxed{\frac{1 + HRs}{\sqrt{1 - s^2}} = -\xi R\,, \qquad \frac{p_R}{8\pi M_{\mathrm{p}}^2} = R\,(\mathrm{arctanh}(s) + HR)}$$

At large R, constraint $\Longrightarrow$ boosted CDL trajectories

At smaller R, branch choices fixed by continuity/tunneling interpretion

BoN Hamiltonian in the late-time limit

$$\frac{S_{\alpha\to\pi}}{4\pi} = 2M_{\mathrm{p}}^2 \int \mathrm{d}t \left[H^2R^3 - R + R\dot{R}\,\mathrm{arctanh}(\dot{R} - HR)\right] - \int \mathrm{d}t R^2\sqrt{1-(\dot{R}-HR)^2}\sigma$$

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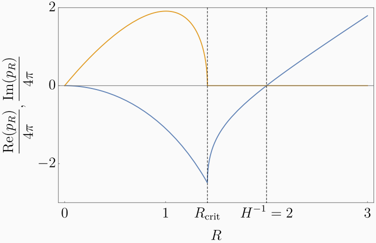
Same rates, different story



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- Type–B: $\mathrm{Im}\, p_R$ stays non-zero beyond R_{crit}
- Nevertheless,

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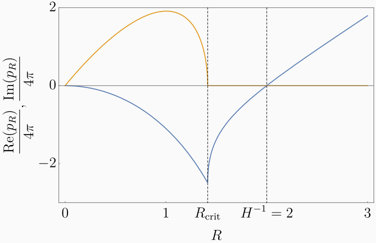

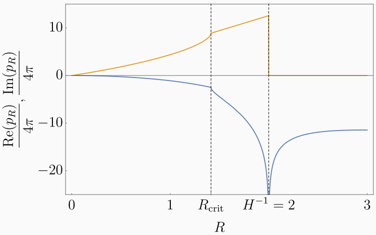

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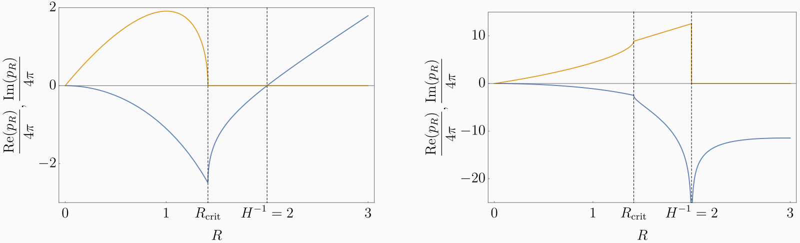

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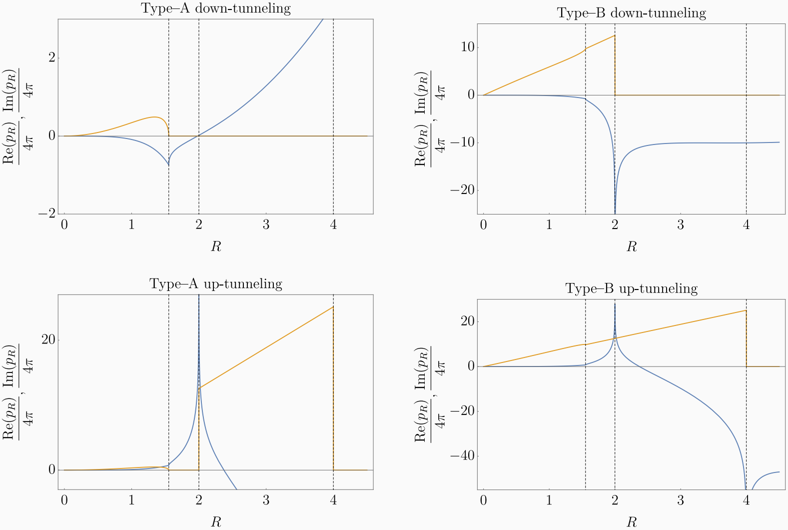
dS–to–dS transitions

- Two-sided version: two Hubble scales H_1, H_2



Conclusions

- CDL rates survive a direct late-time Hamiltonian treatment:
$$B_{\text{late}} = B_{\text{CDL}}$$
- The Lorentzian interpretation changes sharply in the gravity-dominated regime
- For BoN Type–B, up-tunneling, and Type–B dS–to–dS transitions, the bubble goes on shell at the **parent horizon**, not at R_{crit}
- Clear late-time geometric picture

Thank you for your attention!

Outlooks

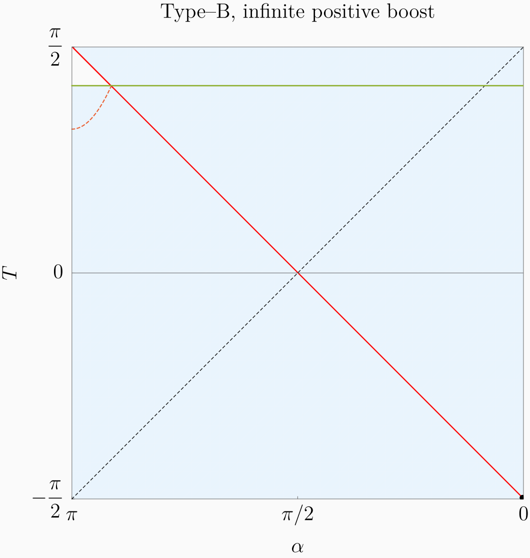
- Staying at finite time $\longrightarrow \Psi(R) \to \Psi(a, R)$

- The **Larfors–Johnson problem is reformulated**: Unstable domain walls require their late-time rate computation

- Wall coupled to bulk scalar field to describe decay

- Deep reason why CDL comes out?

On-shell at the horizon



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$$+ \int \mathrm{d}t_1 2M_{\mathrm{p}}^2 \left[- H_2 R^2 \dot{R} + \eta_1 R \sqrt{(1 - H_2^2 R^2)\left[1 - H_1 R(H_1 R - 2\dot{R})\right] + H_2^2 R^2 \dot{R}^2} \right.$$

$$\left. + R\dot{R}\,\mathrm{arctanh}\left(H_2 R + \frac{\dot{R}(1 - H_2^2 R^2)}{H_2 R\dot{R} - \eta_1 \sqrt{(1 - H_2^2 R^2)\left[1 - H_1 R(H_1 R - 2\dot{R})\right] + H_2^2 R^2 \dot{R}^2}} \right) \right]$$

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