# Vacuum stability in open string models with broken supersymmetry

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Work in progress in collaboration with S. Abel[2], E. Dudas[1] and H. Partouche[1]

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# Introduction

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- Phenomenologically, supersymmetry (SUSY) must be broken
- Generically, classical flat space is destabilized

$\rightarrow \mathcal{V}_{\text{1-loop}} \sim M_{\text{s}}^d$ if hard breaking

$\rightarrow$ If spontaneous breaking at scale $M$ in classically flat space

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No-scale model [Cremmer, Ferrara, Kounnas, Nanopoulos,'83]

$\mathcal{V}_{\text{tree}}$ is independent of $M$

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Unless in specific cases, **still too high**

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## Purpose of the study

- We work in type I strings compactified on $T^2 \times T^4/\mathbb{Z}_2$ with spontaneously broken supersymmetry ($\mathcal{N} = 2 \to 0$)
- Breaking induced by a stringy Scherk-Schwarz mechanism
  $\to$ SUSY breaking scale $M = \frac{1}{2R}$
- At one-loop, we want to have a **positive potential** and try to lower its order of magnitude
- We address the question of **stability**

  $\to$ Tadpoles

  $\to$ Tachyonic moduli
- All this is a follow-up of a study done in $d$-dimensions with $\mathcal{N} = 4 \to 0$ [Abel, Dudas, Lewis, Partouche,'18]

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## Review of the $\mathcal{N} = 4$ case

The potential is given by $\mathcal{V}_{\text{1-loop}} = -\frac{M_{\mathsf{s}}^d}{2(2\pi)^d} \left(\mathcal{T} + \mathcal{K} + \mathcal{A} + \mathcal{M}\right)$

$$\boxed{\mathcal{V}_{\text{1-loop}} \propto \int \mathrm{d}\tau_2 \, \mathrm{Str} \, e^{-\pi\tau_2 m^2}}$$

$\rightarrow$ The lightest states produces the dominant contribution

$\Rightarrow$ Up to exponentially suppressed terms, **if there is no mass scale lower than the SUSY breaking scale** $M = \frac{1}{2R}$

$$\boxed{\mathcal{V}_{\text{1-loop}} = (n_{\mathsf{F}} - n_{\mathsf{B}})\xi M^d + \mathcal{O}\left((cM_{\mathsf{s}}M)^{\frac{d}{2}} e^{-cM_{\mathsf{s}}/M}\right)}$$

with $\xi > 0$ and $cM_{\mathsf{s}}$ a large scale. $n_{\mathsf{F}}$ and $n_{\mathsf{B}}$ count the fermionic and bosonic massless degrees of freedom

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Now introduce Wilson lines (WL) $a_r^I$ along the $I$-th compactified circle for the $r$-th Cartan

If the WL do not introduce a mass scale lower than $M$, then the potential reads [Kounnas, Partouche,'16][Coudarchet, Partouche,'18]

$$\boxed{\mathcal{V}_{\text{1-loop}} = (n_{\mathsf{F}} - n_{\mathsf{B}})\xi M^d + \#(T_{\mathcal{R}_{\mathsf{B}}} - T_{\mathcal{R}_{\mathsf{F}}})(a_r^I)^2 + \cdots}$$

with $\# > 0$ and $T_{\mathcal{R}_{\mathsf{B}}}$ and $T_{\mathcal{R}_{\mathsf{F}}}$ are the Dynkin indices of the representations $\mathcal{R}_{\mathsf{B}}$ and $\mathcal{R}_{\mathsf{F}}$ in which the bosons and fermions live

- no mass scale below $M$ ensures no linear term $\Rightarrow$ no tadpole
- **"life is not easy": the more positive the potential is, the more tachyonic it tends to be and conversely**

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The goal is to find models without tachyons and with $n_{\mathsf{F}} = n_{\mathsf{B}}$

$\rightarrow$ Super no-scale models [Kounnas, Partouche,'15]

In the $\mathcal{N} = 4$ model:

- compactification on $T^{10-d}$
- Scherk-Schwarz mechanism along the ninth direction

It is useful to visualize things in the type I' theory, $T$-dualized along the internal torus

- $2^{10-d}$ $O(d-1)$-planes located at the corners of the **"internal box"**
- $32$ $D(d-1)$-branes are located on the $O$-planes to ensure the absence of tadpoles

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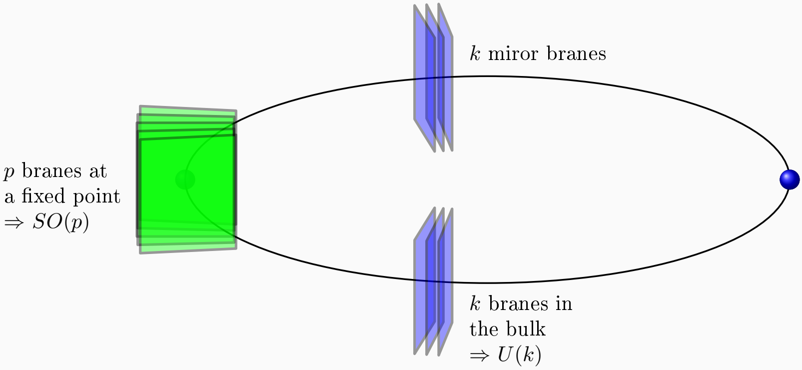


$\rightarrow$ a single brane at a fixed point is frozen

$\rightarrow$ produces a trivial group factor schematically written $SO(1)$

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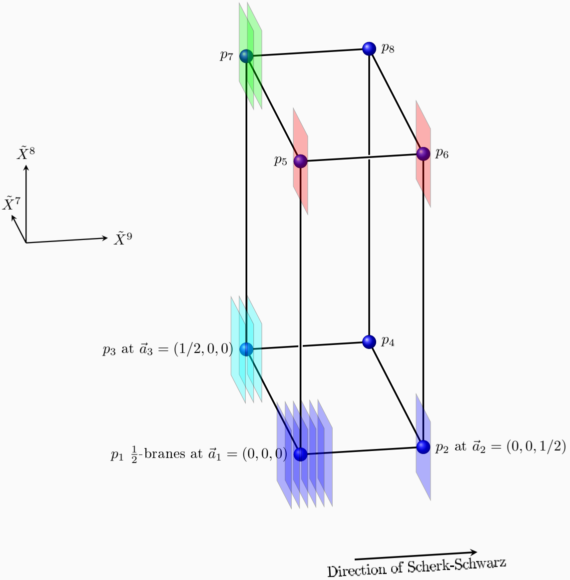

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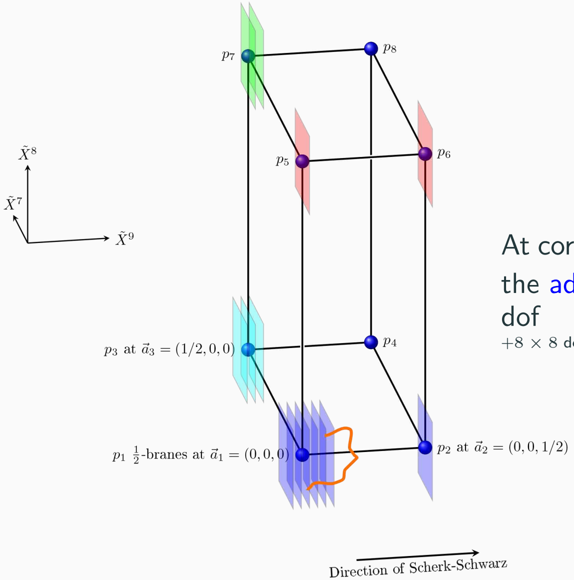


At corner $p_A$, 8 bosons in the adjoint $\Rightarrow 8\frac{p_A(p_A-1)}{2}$ dof

$+8 \times 8$ dof from the closed string sector

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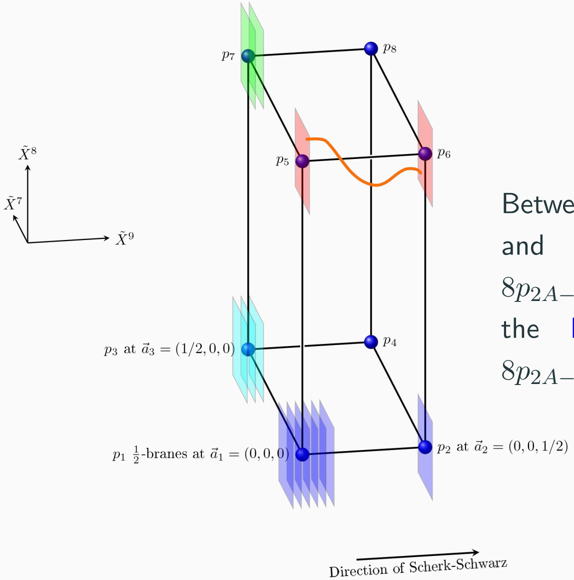


Between corners $p_{2A-1}$ and $p_{2A}$, $8p_{2A}$ and $8p_{2A-1}$ fermions in the bifundamental $\Rightarrow$ $8p_{2A-1}p_{2A}$ dof

## Review of the $\mathcal{N} = 4$ case

It is all the information needed to find stable and super no-scale configurations

- **A lot of models** have a negative potential with $(n_\mathsf{F} - n_\mathsf{B}) < 0$
- Only **a few** have $(n_\mathsf{F} - n_\mathsf{B}) = 0$
    - Enough $O$-planes must be present $\Rightarrow d \leq 5$
    - Configurations with gauge groups up to $SO(5)$
- Possible to reach $(n_\mathsf{F} - n_\mathsf{B}) = 64$ with no gauge group
- Closed string moduli are flat directions

**All these algebraic computations can be recovered from an explicit computation of the potential**

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# $\mathcal{N} = 2 \to 0$ model

## Setup

We start from the Gimon-Polchinski-Pradisi-Sagnotti model

- Compactification on $T^2 \times T^4/\mathbb{Z}_2$, directions $(4,5,6,7,8,9)$
- The circle in the $T^2$ in direction $5$ is used to implement the Scherk-Schwarz breaking

The RR tadpole cancellation condition requires:

- $32$ $D9$-branes $\to N$
- $32$ $D5$-branes orthogonal to the $T^4 \to D$

No vectors running in the Möbius partition function $\Rightarrow$ unitary representations

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# Setup

- We add discrete Wilson lines for the $D9$-branes and for the orthogonal part of the $D5$-branes
- We add discrete positions for the $D5$-branes inside the $T^4$
$\rightarrow$ they are at corners of the internal box

**All discrete Wilson lines can be seen as discrete positions in the correct $T$-dual picture**

Corner labelling: In total, $2^4 \times 2^2 = 64$ corners

Label by $ii'$, where $\rightarrow i = 1, \ldots, 16$ corresponds to the $T^4$
$\rightarrow i' = 1, \ldots, 4$ corresponds to the $T^2$

$(2i' - 1)$ and $2i'$ are opposite corners along the Scherk-Schwarz direction

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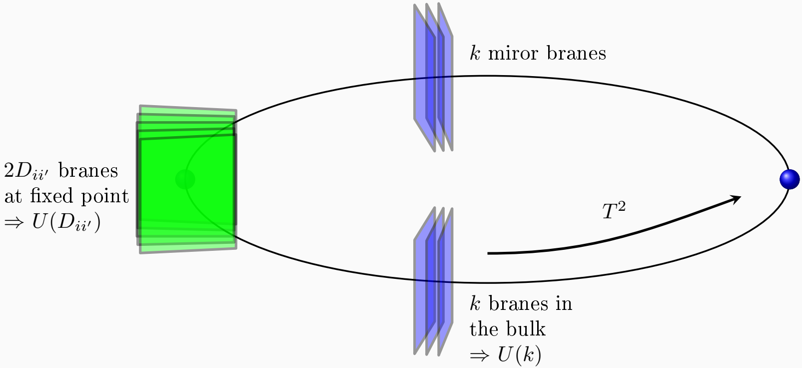
# Dynamical branes

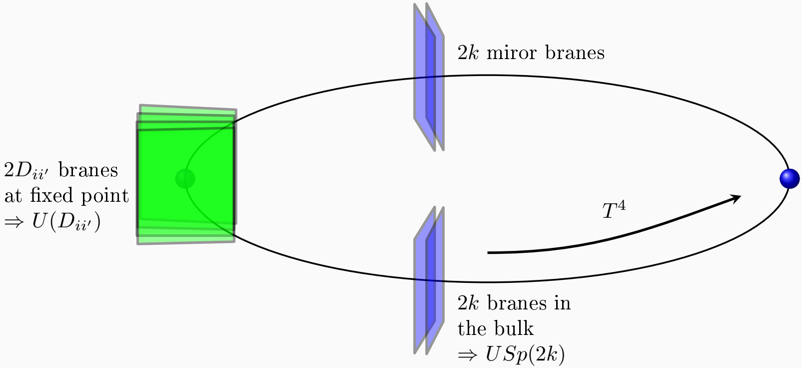
Along the $T^2$:




$\rightarrow$ branes cannot be frozen anymore because the group must be unitary

# Dynamical branes

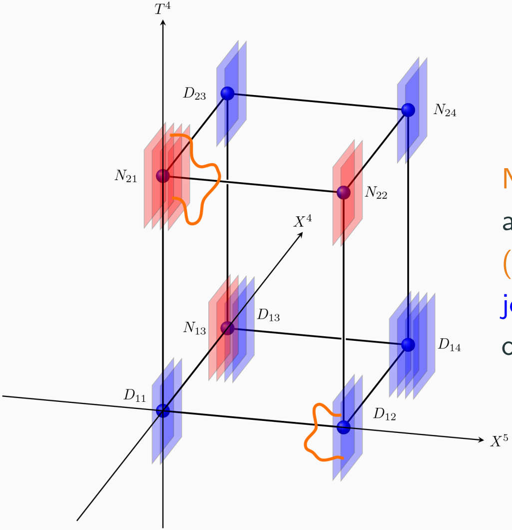
Along the $T^4$: The orbifold cuts again half of the dof




→ Branes are always in pairs $\Rightarrow N_{ii'} = 2n_{ii'}$ and $D_{ii'} = 2d_{ii'}$

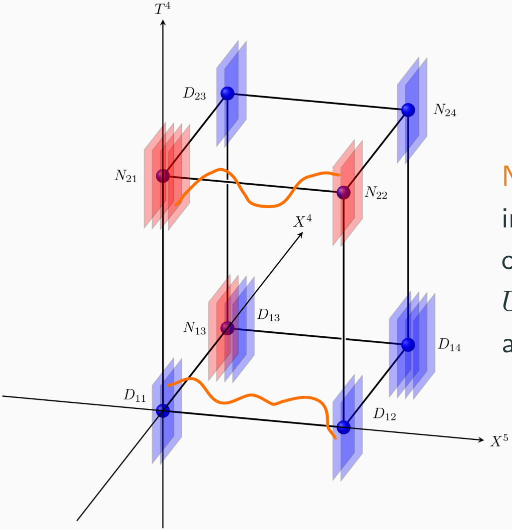
→ They can only move by multiple of four

# Massless spectrum



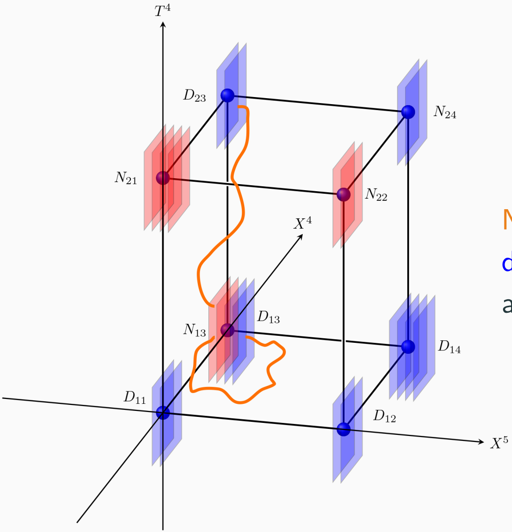

Neuman-Neuman (NN) and Dirichlet-Dirichlet (DD) bosons in the adjoint and antisymmetric of $U(n_{ii'})$ and $U(d_{ii'})$

# Massless spectrum




NN and DD fermions in the bifundamental of $U(n_{i(2i'-1)})$ and $U(n_{i(2i')})$ or $U(d_{i(2i'-1)})$ and $U(d_{i(2i')})$

# Massless spectrum

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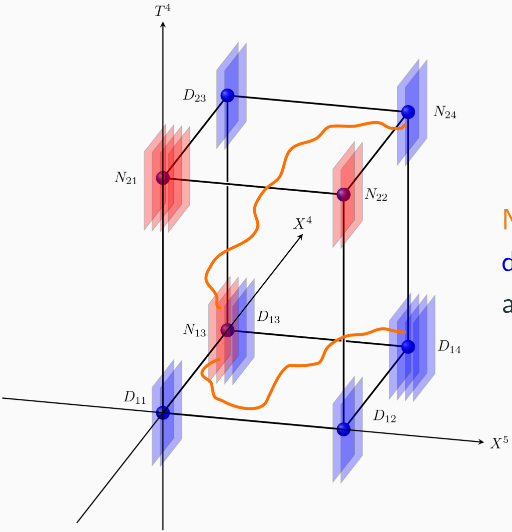


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# Massless spectrum

From the partition function, in total we count in the open string sector:

$$n_{\mathsf{B}}^{\mathsf{open}} = 8\left(n_{ii'}^2 + d_{ii'}^2 + \frac{1}{2} n_{ii'} d_{ji'} - 16\right)$$

$$n_{\mathsf{F}}^{\mathsf{open}} = 8\Big(n_{i(2i'-1)} n_{i(2i')} + d_{i(2i'-1)} d_{i(2i')} + \frac{1}{2} n_{i(2i'-1)} d_{j(2i')} + n_{i(2i)} d_{j(2i'-1)}\Big)$$

The closed string spectrum is the bosonic content (because **fermions are massive**) of the $T^4/\mathbb{Z}_2$ orientifold which is in six dimensions:

- 1 gravity multiplet $g_{MN}$, $B^+_{MN}$
- 1 tensor multiplet $B^-_{MN}$, $\phi$ $\longrightarrow$ 92 degrees of freedom
- 20 hypermultiplets $20 \times 4\phi$

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# Super no-scale and stability conditions

Super no-scale:

$$\left(n_{i(2i'-1)} - n_{i(2i')}\right)^2 + \left(d_{i(2i'-1)} - d_{i(2i')}\right)^2 + \frac{1}{2}\left(n_{i(2i'-1)} - n_{i(2i')}\right)\left(d_{j(2i'-1)} - d_{j(2i')}\right) = 4$$

Stability

$$\mathcal{N} = 4 \to 0 \xrightarrow{\text{orbifold}} \mathcal{N} = 2 \to 0$$

Vector multiplet → Vector multiplet

Vector multiplet → Hypermultiplet

Vector multiplet → Twisted sector

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$$\begin{tikzcd} & \text{Vector multiplet} \\ \text{Vector multiplet} \arrow[ur] \arrow[r] \arrow[dr] & \text{Hypermultiplet} \\ & \text{Twisted sector} \end{tikzcd}$$

# Super no-scale and stability conditions

$$\text{Moduli space} = \underbrace{\text{Coulomb branch} \times \text{Higgs branch}}_{\text{Wilson lines}} \times \text{Twisted scalars}$$

$$\times \underbrace{\text{Closed string moduli}}_{\text{WL, flat directions}}$$

Dynkin indices:

| | Representation $\mathcal{R}$ | $\mathcal{T}_{\mathcal{R}}$ |
|---|---|---|
| $SU(q)$, $q \geq 2$ | fundamental | 1 |
| | $\overline{\text{fundamental}}$ | 1 |
| | adjoint | $2q$ |
| | antisymmetric | $q-2$ |
| | $\overline{\text{antisymmetric}}$ | $q-2$ |

# Super no-scale and stability conditions

Moduli space $= \underbrace{\text{Coulomb branch} \times \text{Higgs branch}}_{\text{Wilson lines}} \times$ Twisted scalars

$\times \underbrace{\text{Closed string moduli}}_{\text{WL, flat directions}}$

Dynkin indices:

| | Representation $\mathcal{R}$ | $\mathcal{T}_{\mathcal{R}}$ |
|---|---|---|
| $SU(q)$, $q \geq 2$ | fundamental | $1$ |
| | $\overline{\text{fundamental}}$ | $1$ |
| | adjoint | $2q$ |
| | antisymmetric | $q-2$ |
| | $\overline{\text{antisymmetric}}$ | $q-2$ |

## Super no-scale and stability conditions

Example for $U(n_{i(2i')})$

Bosons:

- $4$ in the adjoint
- $4$ in the antisymmetric
- $4$ in the $\overline{\text{antisymmectric}}$
- $2\sum_j d_{j(2i')}$ in the fund
- $2\sum_j d_{j(2i')}$ in the $\overline{\text{fund}}$

Fermions:

- $2\times 4n_{i(2i'-1)}$ in the fund
- $2\times 4n_{i(2i'-1)}$ in the $\overline{\text{fund}}$
- $2\sum_j d_{j(2i'-1)}$ in the fund
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- We can write the same thing for $U(n_{i(2i'-1)})$
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# Super no-scale and stability conditions

Higgs branch:

$$n_{i(2i'-1)} - n_{i(2i')} - 1 \geq 0 \quad \text{for} \quad n_{i(2i'-1)} \geq 2$$

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Coulomb branch:

$$4(n_{i(2i')} - n_{i(2i'-1)}) + \sum_{j=1}^{16}(d_{j(2i')} - d_{j(2i'-1)}) - 4 \geq 0 \quad \text{for} \quad n_{i(2i')} \geq 1$$

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# Explicit computation

Wilson lines for $D9$-branes and $D5$-branes:

$$a_\alpha^{\mathcal{I}} = \langle a_\alpha^{\mathcal{I}} \rangle + \epsilon_\alpha^{\mathcal{I}}, \qquad \langle a_\alpha^{\mathcal{I}} \rangle \in \left\{ 0, \frac{1}{2} \right\}, \quad \alpha = 1, \ldots, 32, \ \mathcal{I} = 4, \ldots, 9$$

$$b_\alpha^{\mathcal{I}} = \langle b_\alpha^{\mathcal{I}} \rangle + \xi_\alpha^{\mathcal{I}}, \qquad \langle b_\alpha^{\mathcal{I}} \rangle \in \left\{ 0, \frac{1}{2} \right\}, \quad \alpha = 1, \ldots, 32, \ \mathcal{I} = 4, \ldots, 9$$

$\mathcal{I}$ is split into $I = 6, \ldots, 9 \longrightarrow$ Higgs branch

and $I' = 4, 5 \longrightarrow$ Coulomb branch

No intermediate mass scale:

$G_{\mathcal{I}\mathcal{J}}$ is the metric of the internal space, $\sqrt{G_{55}} = R$

$$G^{55} \ll |G_{ij}| \ll G_{55}, \quad |G_{5j}| \ll \sqrt{G_{55}}, \quad G_{55} \gg 1 \quad i, j \neq 5$$

$$\boxed{\mathcal{V}_{\text{1-loop}} = \frac{\Gamma\left(\frac{5}{2}\right)}{\pi^{\frac{13}{2}}} M^4 \sum_{l_5} \frac{\mathcal{N}_{2l_5+1}(\epsilon, \xi, G)}{|2l_5+1|^5} + \cdots}$$

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$$
\begin{aligned}
\mathcal{N}_{2l_5+1}(\epsilon,\xi,G) = 2\Bigg\{ &-\sum_{(\alpha,\beta)\in L_N}(-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\epsilon^{I'}_\alpha-\epsilon^{I'}_\beta\right)\right] \\
&\times \mathcal{H}_{\frac{5}{2}}\left(\pi|2l_5+1|\frac{\left[(\epsilon^I_\alpha-\epsilon^I_\beta)G^{IJ}(\epsilon^J_\alpha-\epsilon^J_\beta)+(\epsilon^4_\alpha-\epsilon^4_\beta)^2\hat{G}^{44}\right]^{\frac{1}{2}}}{\sqrt{G^{55}}}\right) \\
&-\sum_{(\alpha,\beta)\in L_D}(-)^F \cos\left[2\pi|2l_5+1|\frac{G^{5I'}}{G^{55}}\left(\xi^{I'}_\alpha-\xi^{I'}_\beta\right)\right] \\
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\end{aligned}
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$L_N, L_D, L_{N-D} \Rightarrow$ massless

$F$, fermion number

$\hat{G}^{44} = G^{44} - \frac{G^{45}}{G^{55}}G^{55}\frac{G^{54}}{G^{55}}$

$\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)}z^\nu K_\nu(2z)$

# Explicit computation

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# Explicit computation

Dynamical degrees of freedom:

$$\epsilon_r^I, \xi_r^I \qquad I = 6, \ldots, 9, \quad \text{and} \quad r = 1, \ldots, \sum_{i=1}^{16} \sum_{i'=1}^{4} \left\lfloor \frac{n_{ii'}}{2} \right\rfloor$$

$$\epsilon_{r'}^{I'}, \xi_{r'}^{I'}, \quad I' = 4, 5, \qquad \text{and} \quad r' = 1, \ldots, 16$$

$$\mathcal{N}_{2l_9+1} = 32\pi^2 (2l_9+1)^2 \Bigg\{ \mathcal{O}\left(\epsilon^0, \xi^0\right) + \sum_r \left( n_{i(r)i'(r)} - n_{i(r)\tilde{i}'(r)} - 1 \right) \epsilon_r^I \Delta^{IJ} \epsilon_r^J$$
$$+ \sum_{r'} \left( n_{i(r')i'(r')} - n_{i(r')\tilde{i}'(r')} - 1 + \frac{1}{4} \sum_i \left( d_{ii'(r')} - d_{i\tilde{i}'(r')} \right) \right) \epsilon_{r'}^{I'} \Delta^{I'J'} \epsilon_{r'}^{J'}$$
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# Explicit computation

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## Twisted scalars

How to find the mass of the twisted scalars?

- The twisted scalars are not Wilson lines
- They are Neuman-Dirichlet states

Idea:

- A **string computation** of their one-loop two-point function
  - Tedious, requires the technology of twist fields and their correlators [Atick, Dixon, Griffin, Nemeschansky,'87]
  - Extracting only the sign of the mass is maybe doable

$\rightarrow$ maybe a field theory approach would be simpler

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  - Tedious, requires the technology of twist fields and their correlators [Atick, Dixon, Griffin, Nemeschansky,'87]
  - Extracting only the sign of the mass is maybe doable

→ maybe a field theory approach would be simpler

# Twisted scalars

How to find the mass of the twisted scalars?

- The twisted scalars are not Wilson lines
- They are Neuman-Dirichlet states

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- We expressed the super no-scale condition *via* the counting of massless degrees of freedom
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**Thank you for your attention!**

# Annulus partition function with discrete WL

$$
\begin{aligned}
\mathcal{A} = \frac{1}{4}\int_0^\infty \frac{\mathsf{d}\tau_2}{\tau_2^3}\Bigg\{ \Bigg[ &(V_4O_4 + O_4V_4)\left( N_{ii'}N_{jj'}\,\frac{P^{(4)}_{\vec{m}+\vec{a}_i-\vec{a}_j}}{\eta^4} + D_{ii'}D_{jj'}\,\frac{W^{(4)}_{\vec{n}+\vec{a}_i-\vec{a}_j}}{\eta^4} \right) \\
&+ (V_4O_4 - O_4V_4)\left(R^N_{ii'}R^N_{ij'} + R^D_{ii'}R^D_{ij'}\right)\left(\frac{2\eta}{\theta_2}\right)^2 + 2N_{ii'}D_{jj'}(O_4C_4 + V_4S_4)\left(\frac{\eta}{\theta_4}\right)^2 \\
&+ 2e^{4i\pi\vec{a}_i\cdot\vec{a}_j}R^N_{ii'}R^D_{jj'}(O_4C_4 - V_4S_4)\left(\frac{\eta}{\theta_3}\right)^2\Bigg]\frac{P^{(2)}_{\vec{m}'+\vec{a}_{i'}-\vec{a}_{j'}}}{\eta^4} \\
&- \Bigg[(S_4S_4 + C_4C_4)\left( N_{ii'}N_{jj'}\,\frac{P^{(4)}_{\vec{m}+\vec{a}_i-\vec{a}_j}}{\eta^4} + D_{ii'}D_{jj'}\,\frac{W^{(4)}_{\vec{n}+\vec{a}_i-\vec{a}_j}}{\eta^4} \right) \\
&+ (C_4C_4 - S_4S_4)\left(R^N_{ii'}R^N_{ij'} + R^D_{ii'}R^D_{ij'}\right)\left(\frac{2\eta}{\theta_2}\right)^2 + 2N_{ii'}D_{jj'}(S_4O_4 + C_4V_4)\left(\frac{\eta}{\theta_4}\right)^2 \\
&+ 2e^{4i\pi\vec{a}_i\cdot\vec{a}_j}R^N_{ii'}R^D_{jj'}(S_4O_4 - C_4V_4)\left(\frac{\eta}{\theta_3}\right)^2\Bigg]\frac{P^{(2)}_{\vec{m}'+\vec{a}'_{\mathsf{s}}+\vec{a}_{i'}-\vec{a}_{j'}}}{\eta^4}\Bigg\}
\end{aligned}
$$

# Möbius partition function with discrete WL

$$
\begin{aligned}
\mathcal{M} = -\frac{1}{4}\int_0^\infty \frac{\mathsf{d}\tau_2}{\tau_2^3}\Bigg\{ \Bigg[ &(\hat{V}_4\hat{O}_4 + \hat{O}_4\hat{V}_4)\left( N_{ii'}\frac{P^{(4)}_{\vec{m}}}{\hat{\eta}^4} + D_{ii'}\frac{W^{(4)}_{\vec{n}}}{\hat{\eta}^4} \right) \\
&- (N_{ii'} + D_{ii'})(\hat{V}_4\hat{O}_4 - \hat{O}_4\hat{V}_4)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \Bigg] \frac{P^{(2)}_{\vec{m}'}}{\hat{\eta}^4} \\
&- \Bigg[ (\hat{C}_4\hat{C}_4 + \hat{S}_4\hat{S}_4)\left( N_{ii'}\frac{P^{(4)}_{\vec{m}}}{\hat{\eta}^4} + D_{ii'}\frac{W^{(4)}_{\vec{n}}}{\hat{\eta}^4} \right) \\
&- (N_{ii'} + D_{ii'})(\hat{C}_4\hat{C}_4 - \hat{S}_4\hat{S}_4)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \Bigg] \frac{P^{(2)}_{\vec{m}'+\vec{a}'_{\mathsf{s}}}}{\hat{\eta}^4} \Bigg\}
\end{aligned}
$$

# Massless spectrum

## Bosons:

$$V_4O_4\left[n_{ii'}\bar{n}_{ii'}+d_{ii'}\bar{d}_{ii'}\right]$$

$$+O_4V_4\left[\frac{n_{ii'}(n_{ii'}-1)}{2}+\frac{\bar{n}_{ii'}(\bar{n}_{ii'}-1)}{2}+\frac{d_{ii'}(d_{ii'}-1)}{2}+\frac{\bar{d}_{ii'}(\bar{d}_{ii'}-1)}{2}\right]$$

$$+\frac{O_4C_4}{2}\left[\left(1-e^{4i\pi\vec{a}_i\cdot\vec{a}_j}\right)\left(n_{ii'}d_{ji'}+\bar{n}_{ii'}\bar{d}_{ji'}\right)+\left(1+e^{4i\pi\vec{a}_i\cdot\vec{a}_j}\right)\left(n_{ii'}\bar{d}_{ji'}+\bar{n}_{ii'}d_{ji'}\right)\right]$$

## Fermions:

$$C_4C_4\left[n_{i(2i'-1)}\bar{n}_{i(2i')}+\bar{n}_{i(2i'-1)}n_{i(2i')}+d_{i(2i'-1)}\bar{d}_{i(2i')}+\bar{d}_{i(2i'-1)}d_{i(2i')}\right]$$

$$+S_4S_4\left[n_{i(2i'-1)}n_{i(2i')}+\bar{n}_{i(2i'-1)}\bar{n}_{i(2i')}+d_{i(2i'-1)}d_{i(2i')}+\bar{d}_{i(2i'-1)}\bar{d}_{i(2i')}\right]$$

$$+\frac{S_4O_4}{2}\Big[\left(1-e^{4i\pi\vec{a}_i\cdot\vec{a}_j}\right)\left(n_{i(2i'-1)}d_{j(2i')}+n_{i(2i')}d_{j(2i'-1)}+\bar{n}_{i(2i'-1)}\bar{d}_{j(2i')}+\bar{n}_{i(2i')}\bar{d}_{j(2i'-1)}\right)$$

$$+\left(1+e^{4i\pi\vec{a}_i\cdot\vec{a}_j}\right)\left(n_{i(2i'-1)}\bar{d}_{j(2i')}+n_{i(2i')}\bar{d}_{j(2i'-1)}+\bar{n}_{i(2i'-1)}d_{j(2i')}+\bar{n}_{i(2i')}d_{j(2i'-1)}\right)\Big]$$